Neutrinoless double-β decay with three or four neutrino mixing

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Abstract

Considering the scheme with mixing of three neutrinos and a mass hierarchy that can accommodate the results of solar and atmospheric neutrino experiments, it is shown that the results of solar neutrino experiments imply a lower bound for the effective Majorana mass in neutrinoless double-β decay, under the natural assumptions that massive neutrinos are Majorana particles and there are no unlikely fine-tuned cancellations among the contributions of the different neutrino masses. Considering the four-neutrino schemes that can accommodate also the results of the LSND experiment, it is shown that only one of them is compatible with the results of neutrinoless double-β decay experiments and with the measurement of the abundances of primordial elements produced in Big-Bang Nucleosynthesis. It is shown that in this scheme, under the assumptions that massive neutrinos are Majorana particles and there are no cancellations among the contributions of the different neutrino masses, the results of the LSND experiment imply a lower bound for the effective Majorana mass in neutrinoless double-β decay.

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I. INTRODUCTION

Neutrino oscillations [1–3] is one of the most intriguing phenomena of today high-energy physics and one of the most promising ways to explore the physics beyond the Standard Model. At present there are three experimental indications in favor of neutrino oscillations that have been obtained in solar neutrino experiments (Homestake [4], Kamiokande [5], GALLEX [6], SAGE [7] and Super-Kamiokande [8]), in atmospheric neutrino experiments (Kamiokande [9], IMB [10], Super-Kamiokande [11], Soudan-2 [12] and MACRO [13])) and in the accelerator LSND experiment [14,15]. On the other hand, neutrinoless double-β decay $(\beta\beta_{0\nu})$ experiments [16–19] and the experiments on the direct measurement of neutrino masses [20] did not obtain any positive result so far (see [21]). The connection between the properties of neutrinos that determine neutrino oscillations (mass squared differences and neutrino mixing) and neutrinoless double-β decay have been discussed in many papers [22–33]. In this paper we discuss some implications of the latest results of neutrino oscillation experiments for neutrinoless double-β decay and we show that under reasonable assumptions there is a lower bound for the effective Majorana neutrino mass measured in $\beta\beta_{0\nu}$ decay experiments.

If massive neutrinos are Majorana particles, the matrix element of $\beta\beta_{0\nu}$ decay is proportional to the effective Majorana mass

$$|\langle m \rangle| = \left| \sum_k U_{ek}^2 \, m_k \right| , \tag{1.1}$$

where U is the mixing matrix that connects the flavor neutrino fields $\nu_{\alpha L}$ ($\alpha = e, \mu, \tau$) to the fields ν_{kL} of neutrinos with masses m_k through the relation

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \, \nu_{kL} . \tag{1.2}$$

The present experimental upper limit for $|\langle m \rangle|$,

$$|\langle m \rangle|_{\exp} \leq 0.2 - 0.4 \, \text{eV} \qquad (90\% \text{ CL}) , \tag{1.3}$$

has been obtained from the measurement of the half-life of ^{76}Ge in the Heidelberg–Moscow experiment ($T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 5.7 \times 10^{25}$ yr at 90% CL) [18]. The range of the upper bound (1.3) is due to the uncertainty of the theoretical calculation of the nuclear matrix element and has been obtained from the results of different calculations using the Quasiparticle Random Phase Approximation (QRPA) [34,35]. In particular, the recent QRPA calculation in [35] yields the rather stringent upper bound $|\langle m \rangle|_{\exp} \leq 0.27$ eV. On the other hand, the Shell Model calculation in [36] yields the loser bound $|\langle m \rangle|_{\exp} \leq 0.56$ eV. However, the calculation of the nuclear matrix element for the neutrinoless double-β decay of ^{76}Ge presented in [36] has been truncated before reaching convergence and the full calculation is expected to yield a more stringent upper bound for $|\langle m \rangle|_{\exp}$. Therefore, in the following we will consider the range in Eq. (1.3) as a reliable estimate of the uncertainty of the experimental upper bound for the effective Majorana mass $|\langle m \rangle|$.

The next generation of $\beta\beta_{0\nu}$ decay experiments is expected to be sensitive to values of $|\langle m \rangle|$ in the range $10^{-2} - 10^{-1}$ eV [17]. Values of $|\langle m \rangle|$ as small as about 10^{-3} eV may be reachable not far in the future [19].

After the measurement in the Super-Kamiokande experiment of an up-down asymmetry of μ-like events induced by atmospheric muon neutrinos, the experimental evidence in favor of oscillations of atmospheric neutrinos is widely considered to be beyond reasonable doubts (see, for example, [2,37,3]). There are also convincing arguments in favor of a neutrino oscillation explanation of the solar neutrino problem (see, for example, [38,2,39]). Therefore, in this paper we will consider first, in Section II, the implications for $\beta\beta_{0\nu}$ decay in the scheme with mixing of three neutrinos and a mass hierarchy that can accommodate the results of atmospheric and solar neutrino experiments. In Section III we consider the schemes with four massive neutrinos that can accommodate all neutrino oscillation data, including the LSND results in favor of $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations that wait for independent confirmations by other experiments [40,41].

II. THREE NEUTRINOS WITH A MASS HIERARCHY

The results of solar and atmospheric neutrino experiments indicate the existence of a hierarchy of mass-squared differences (see [38,42–45,11]):

$$\Delta m^2_{\text{sun}} \lesssim 10^{-4}\,\text{eV}^2 \ll 10^{-3}\,\text{eV}^2 \lesssim \Delta m^2_{\text{atm}} \lesssim 10^{-2}\,\text{eV}^2 \,, \tag{2.1}$$

where Δm^2_{sun} and Δm^2_{atm} are the mass-squared differences relevant for solar and atmospheric neutrino oscillations, respectively. A natural scheme that can accommodate this hierarchy is the one with three neutrinos and a mass hierarchy,

$$\overbrace{m_1 \ll m_2}^{\text{sun}} \underbrace{\ll m_3}_{\text{atm}} \,. \tag{2.2}$$

In the framework of the hierarchical spectrum (2.2) the mass-squared differences relevant for the oscillations of solar and atmospheric neutrinos are

$$\Delta m^2_{\text{sun}} = \Delta m^2_{21} \equiv m_2^2 - m_1^2 \simeq m_2^2 \,, \qquad \Delta m^2_{\text{atm}} = \Delta m^2_{31} \equiv m_3^2 - m_1^2 \simeq m_3^2 \,. \tag{2.3}$$

The mass hierarchy (2.2) is predicted by the see-saw mechanism [46], which predicts also that the tree light massive neutrinos are Majorana particles. In this case neutrinoless double-β decay is possible.

It has been shown in [25–27] that the results of neutrino oscillation experiments imply a rather stringent upper bound (about 6×10^{-3} eV) for the effective Majorana mass in neutrinoless double-β decay in the scheme with mixing of three neutrinos and a mass hierarchy. In principle the effective Majorana mass (1.1) can be vanishingly small because of cancellations among the contributions of the different mass eigenstates. However, since the neutrino masses and the elements of the neutrino mixing matrix are independent quantities, if there is a hierarchy of neutrino masses such a cancellation would be the result of an unlikely fine-tuning, unless some unknown symmetry is at work. Here we consider the possibility that no

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such symmetry exist and *no unlikely fine-tuning operates to suppress the effective Majorana mass* (1.1). In this case we have

$$|\langle m \rangle| \simeq \max_k |U_{ek}|^2 \, m_k \, . \tag{2.4}$$

Let us define the absolute value of the contribution of the neutrino mass m_k to $|\langle m \rangle|$ as

$$|\langle m \rangle|_k \equiv |U_{ek}|^2 \, m_k \, . \tag{2.5}$$

In the following we will estimate the value of $|\langle m \rangle|$ using the largest $|\langle m \rangle|_k$ obtained from the results of neutrino oscillation experiments.

The results of the CHOOZ experiment [47] and the Super-Kamiokande atmospheric neutrino data [11] imply that $|U_{e3}|^2$ is small ($|U_{e3}|^2 \lesssim 5 \times 10^{-2}$) and there is an upper limit of about 6×10^{-3} eV for the contribution $|\langle m \rangle|_3$ to the effective Majorana mass in $\beta\beta_{0\nu}$ decay [25–27]. Since there is no lower bound for $|U_{e3}|^2$ from experimental data, $|\langle m \rangle|_3$ could be very small.

Hence, the largest contribution to $|\langle m \rangle|$ could come from $|\langle m \rangle|_2 \equiv |U_{e2}|^2 \, m_2$. Since in the framework of the scheme with mixing of three neutrinos and a mass hierarchy $\Delta m^2_{\mathrm{sun}} \simeq m_2^2$ and $|U_{e2}|^2 \simeq \frac{1}{2}\left(1 - \sqrt{1 - \sin^2 2\vartheta_{\mathrm{sun}}}\right)$ [48], where ϑ_{sun} is the two-neutrino mixing angle used in the analysis of solar neutrino data, we have

$$|\langle m \rangle|_2 \simeq \frac{1}{2}\left(1 - \sqrt{1 - \sin^2 2\vartheta_{\mathrm{sun}}}\right) \sqrt{\Delta m^2_{\mathrm{sun}}} \, . \tag{2.6}$$

Solar neutrino data imply bounds for $\sin^2 2\vartheta_{\mathrm{sun}}$ and $\Delta m^2_{\mathrm{sun}}$. In particular the large mixing angle MSW [49] solution (LMA-MSW) of the solar neutrino problem, which seems to be favored by recent data [50], implies that [42]

$$1.2 \times 10^{-5}\,\mathrm{eV}^2 \le \Delta m^2_{\mathrm{sun}} \le 3.1 \times 10^{-4}\,\mathrm{eV}^2 \, , \qquad 0.58 \le \sin^2 2\vartheta_{\mathrm{sun}} \le 1.00 \tag{2.7}$$

at 99% CL, taking into account the total rates measured in solar neutrino experiments and the day-night variations observed in the Super-Kamiokande experiment [8]. Hence, for the contribution of m_2 to the effective Majorana mass we obtain

$$6 \times 10^{-4}\,\mathrm{eV} \lesssim |\langle m \rangle|_2 \lesssim 9 \times 10^{-3}\,\mathrm{eV} \, . \tag{2.8}$$

This estimate does not take into account the correlation between $\Delta m^2_{\mathrm{sun}}$ and $\sin^2 2\vartheta_{\mathrm{sun}}$. The precise allowed range for $|\langle m \rangle|_2$ as a function of $\Delta m^2_{\mathrm{sun}}$ obtained with Eq. (2.6) from the LMA-MSW region (99% CL) in Fig. 2 of Ref. [42] is shown in Fig. 1. The dashed line in Fig. 1 represents the unitarity limit $|\langle m \rangle|_2 \le \sqrt{\Delta m^2_{\mathrm{sun}}}$. From Fig. 1 one can see that the LMA-MSW solution of the solar neutrino problem implies that[1]

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[1] The upper limit $|\langle m \rangle|_2 \lesssim 3 \times 10^{-3}\,\mathrm{eV}$ presented in [27] has been obtained from the 90% CL LMA-MSW region in Fig. 8a of Ref. [43], using Eq. (2.6). The 99% CL LMA-MSW region in Fig. 8a of Ref. [43] gives $|\langle m \rangle|_2 \lesssim 6 \times 10^{-3}\,\mathrm{eV}$, in agreement with the upper bound in Eq. (2.9).

$$7.4 \times 10^{-4} \, \text{eV} \lesssim |\langle m \rangle|_2 \lesssim 6.0 \times 10^{-3} \, \text{eV} \,. \tag{2.9}$$

Assuming the absence of fine-tuned cancellations among the contributions of the three neutrino masses to the effective Majorana mass, if $|U_{e3}|^2$ is very small and $|\langle m \rangle|_3 \ll |\langle m \rangle|_2$, from Eqs. (2.4) and (2.9) we obtain

$$7 \times 10^{-4} \, \text{eV} \lesssim |\langle m \rangle| \lesssim 6 \times 10^{-3} \, \text{eV} \,. \tag{2.10}$$

Hence, we see that, *assuming the absence of an unlikely fine-tuned suppression of $|\langle m \rangle|$, the results of solar neutrino experiments give an indication of the value of the effective Majorana mass in $\beta\beta_{0\nu}$ decay, with a lower bound of about $7 \times 10^{-4} \, \text{eV}$ in the case of the LMA-MSW solution of the solar neutrino problem.* This bound is rather small, but values of $|\langle m \rangle|$ of the order of 10^{-3} eV, indicated by the range (2.10), may be measurable in a not too far future [19].

Also the small mixing angle MSW (SMA-MSW) and the vacuum oscillation (VO) solutions of the solar neutrino problem imply allowed ranges for $|\langle m \rangle|_2$, but their values are much smaller than in the case of the LMA-MSW solution. Using the 99% CL allowed regions obtained in [38] from the analysis of the total rates measured in solar neutrino experiments we have

$$5 \times 10^{-7} \, \text{eV} \lesssim |\langle m \rangle|_2 \lesssim 10^{-5} \, \text{eV} \qquad \text{(SMA-MSW)} \,, \tag{2.11}$$

$$10^{-6} \, \text{eV} \lesssim |\langle m \rangle|_2 \lesssim 2 \times 10^{-5} \, \text{eV} \qquad \text{(VO)} \,. \tag{2.12}$$

III. FOUR NEUTRINOS

If, besides to the solar and atmospheric neutrino data, also the results of the accelerator LSND experiment are taken into account, at least three independent mass-squared differences are needed. This can be seen by considering the general expression for the probability of $\nu_\alpha \to \nu_\beta$ transitions in vacuum [1–3], that can be written as

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_k U_{\alpha k}^* U_{\beta k} \, \exp\left(-i \frac{\Delta m_{kj}^2 \, L}{2 \, E} \right) \right|^2 \,, \tag{3.1}$$

where $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$, j is any of the mass-eigenstate indices, L is the distance between the neutrino source and detector and E is the neutrino energy. The range of L/E probed by each type of experiment is different: $L/E \gtrsim 10^{10} \, \text{eV}^{-2}$ for solar neutrino experiments, $L/E \sim 10^2 - 10^3 \, \text{eV}^{-2}$ for atmospheric neutrino experiments and $L/E \sim 1 \, \text{eV}^{-2}$ for the LSND experiment. From Eq. (3.1) it is clear that neutrino oscillations occur in an experiment only if there is at least one mass-squared difference Δm_{kj}^2 such that

$$\frac{\Delta m_{kj}^2 \, L}{2 \, E} \gtrsim 0.1 \tag{3.2}$$

(the precise lower bound depends on the sensitivity of the experiment) in a significant part of the energy and source-detector distance intervals of that experiment (if the condition (3.2) is

not satisfied, $P_{\nu_\alpha \to \nu_\beta} \simeq |\sum_k U^*_{\alpha k} U_{\beta k}|^2 = \delta_{\alpha\beta}$). Since the range of L/E probed by the LSND experiment is the smaller one, a mass-squared difference is needed for LSND oscillations:

$$\Delta m^2_{\mathrm{LSND}} \gtrsim 10^{-1}\,\mathrm{eV}^2\,. \tag{3.3}$$

Specifically, the maximum likelihood analysis of the LSND data in terms of two-neutrino oscillations gives [15]

$$0.20\,\mathrm{eV}^2 \leq \Delta m^2_{\mathrm{LSND}} \leq 2.0\,\mathrm{eV}^2\,. \tag{3.4}$$

Furthermore, from Eq. (3.1) it is clear that a dependence of the oscillation probability from the neutrino energy E and the source-detector distance L is observable only if there is at least one mass-squared difference Δm^2_{kj} such that

$$\frac{\Delta m^2_{kj}\, L}{2\, E} \sim 1\,. \tag{3.5}$$

Since a variation of the transition probability as a function of neutrino energy has been observed both in solar and atmospheric neutrino experiments and the range of L/E probed by each type of experiment is different, two more mass-squared differences with different scales are needed:

$$\Delta m^2_{\mathrm{sun}} \sim 10^{-10}\,\mathrm{eV}^2 \qquad (\mathrm{VO})\,, \tag{3.6}$$
$$\Delta m^2_{\mathrm{atm}} \sim 10^{-3} - 10^{-2}\,\mathrm{eV}^2\,. \tag{3.7}$$

The condition (3.6) for the solar mass-squared difference $\Delta m^2_{\mathrm{sun}}$ has been obtained under the assumption of vacuum oscillations (VO). If the disappearance of solar ν_e's is due to the MSW effect [49], the condition

$$\Delta m^2_{\mathrm{sun}} \lesssim 10^{-4}\,\mathrm{eV}^2 \qquad (\mathrm{MSW}) \tag{3.8}$$

must be fulfilled in order to have a resonance in the interior of the sun. Hence, in the MSW case $\Delta m^2_{\mathrm{sun}}$ must be at least one order of magnitude smaller than $\Delta m^2_{\mathrm{atm}}$.

The existence of three different scales of neutrino mass-squared differences[2] imply that at least four light massive neutrinos must exist in nature. Here we consider the schemes

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[2] It is possible to ask if three different scales of neutrino mass-squared differences are needed even if the results of the Homestake solar neutrino experiment [4] is neglected, allowing an energy-independent suppression of the solar ν_e flux. The answer is that still the data cannot be fitted with only two neutrino mass-squared differences because an energy-independent suppression of the solar ν_e flux requires large $\nu_e \to \nu_\mu$ or $\nu_e \to \nu_\tau$ transitions generated by $\Delta m^2_{\mathrm{atm}}$ or $\Delta m^2_{\mathrm{LSND}}$. These transitions are forbidden by the results of the Bugey [51] and CHOOZ [47] $\bar{\nu}_e$ disappearance experiments and by the non-observation of an up-down asymmetry of e-like events in the Super-Kamiokande atmospheric neutrino experiment [11]. I would like to thank S.T. Petcov for useful discussions about this point.

with four light and mixed neutrinos [52–58], which constitute the minimal possibility that allows to explain all the data of neutrino oscillation experiments. In this case, in the flavor basis the three active neutrinos ν_e, ν_μ, ν_τ are accompanied by a sterile neutrino ν_s that does not take part in standard weak interactions.

The existence of four light massive neutrinos is a low-energy manifestation of physics beyond the Standard Model (see, for example, [59]). In most theories beyond the Standard Model neutrinos are naturally Majorana particles and neutrinoless double-β decay is allowed. Therefore, we see that the experimental evidences in favor of neutrino oscillations indicate that neutrinos may be Majorana particles and neutrinoless double-β decay is a concrete possibility.

It has been shown [55] that there are only two schemes with four-neutrino mixing that can accommodate the results of all neutrino oscillation experiments:

$$\text{(A)} \quad \overbrace{m_1 < m_2}^{\text{atm}} < \overbrace{m_3 < m_4}^{\text{sun}}, \qquad \text{(B)} \quad \overbrace{m_1 < m_2}^{\text{sun}} < \overbrace{m_3 < m_4}^{\text{atm}}. \tag{3.9}$$

These two spectra are characterized by the presence of two pairs of close masses separated by a gap of about 1 eV which provides the mass-squared difference $\Delta m^2_{\text{LSND}} = \Delta m^2_{41}$ responsible of the oscillations observed in the LSND experiment. In the scheme A $\Delta m^2_{\text{atm}} = \Delta m^2_{21}$ and $\Delta m^2_{\text{sun}} = \Delta m^2_{43}$, whereas in scheme B $\Delta m^2_{\text{atm}} = \Delta m^2_{43}$ and $\Delta m^2_{\text{sun}} = \Delta m^2_{21}$.

The results of the short-baseline $\bar\nu_e$ disappearance experiment Bugey [51], in which no indication in favor of neutrino oscillations was found, imply that the mixing of ν_e with the two "heavy" neutrinos ν_3 and ν_4 is large in scheme A and small in scheme B [55,2]:

$$1 - \left(|U_{e3}|^2 + |U_{e4}|^2 \right) \lesssim 3 \times 10^{-2} \qquad \text{(A)}, \tag{3.10}$$

$$|U_{e3}|^2 + |U_{e4}|^2 \lesssim 3 \times 10^{-2} \qquad \text{(B)}, \tag{3.11}$$

for Δm^2_{LSND} in the LSND-allowed range (3.4). Therefore, if scheme A is realized in nature the effective Majorana mass in $\beta\beta_{0\nu}$ decay can be as large as $m_3 \simeq m_4 \simeq \sqrt{\Delta m^2_{\text{LSND}}} \simeq 0.45 - 1.4\,\text{eV}$ [24–27]. On the other hand, in scheme B neutrinoless double-β decay is strongly suppressed [25–27]. In the following two subsections we discuss some connections between the results of neutrino oscillation experiments and neutrinoless double-β decay in the schemes A and B.

A. Scheme A

In the four-neutrino scheme A, from Eq. (3.10) and the unitarity of the mixing matrix we have $|U_{e1}|^2 + |U_{e2}|^2 \lesssim 3 \times 10^{-2}$. Therefore, the contribution of the two light masses m_1 and m_2 to the effective Majorana mass in $\beta\beta_{0\nu}$ decay can be neglected and we have

$$|\langle m \rangle| \simeq \left| U^2_{e3}\, m_3 + U^2_{e4}\, m_4 \right|, \tag{3.12}$$

that implies the limits

$$\left| |U_{e3}|^2 m_3 - |U_{e4}|^2 m_4 \right| \lesssim |\langle m \rangle| \lesssim |U_{e3}|^2 m_3 + |U_{e4}|^2 m_4. \tag{3.13}$$

Neglecting the small difference between m_3 and m_4 and taking into account that $m_3 \simeq m_4 \simeq \sqrt{\Delta m^2_{\mathrm{LSND}}}$, we have

$$\left| |U_{e3}|^2 - |U_{e4}|^2 \right| \sqrt{\Delta m^2_{\mathrm{LSND}}} \lesssim |\langle m \rangle| \lesssim \left(|U_{e3}|^2 + |U_{e4}|^2 \right) \sqrt{\Delta m^2_{\mathrm{LSND}}} \,. \tag{3.14}$$

Since the quantity $|U_{e3}|^2 + |U_{e4}|^2$ is large in scheme A (see Eq. (3.10)), we obtain

$$\left| |U_{e3}|^2 - |U_{e4}|^2 \right| \sqrt{\Delta m^2_{\mathrm{LSND}}} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m^2_{\mathrm{LSND}}} \,. \tag{3.15}$$

Furthermore, from the inequality (3.10) one can see that the contribution of the mixing of ν_e with ν_1 and ν_2 to the survival probability of solar electron neutrinos is negligible and $|U_{e3}|^2$ and $|U_{e4}|^2$ are related to the mixing angle ϑ_{sun} obtained from the two-generation fit of solar neutrino experiments by

$$|U_{e3}|^2 \simeq \cos^2 \vartheta_{\mathrm{sun}} \,, \qquad |U_{e4}|^2 \simeq \sin^2 \vartheta_{\mathrm{sun}} \,. \tag{3.16}$$

Hence, the range (3.15) can be written as

$$\sqrt{(1 - \sin^2 2\vartheta_{\mathrm{sun}}) \Delta m^2_{\mathrm{LSND}}} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m^2_{\mathrm{LSND}}} \,, \tag{3.17}$$

Let us emphasize that this allowed range for $|\langle m \rangle|$ in scheme A depends only on the assumption that massive neutrinos are Majorana particles.

In the case of the SMA-MSW solution of the solar neutrino problem (for both $\nu_e \to \nu_\tau$ or $\nu_e \to \nu_s$ transitions) $\sin^2 2\vartheta_{\mathrm{sun}}$ is very small ($\sin^2 2\vartheta_{\mathrm{sun}} \lesssim 10^{-2}$) and we have

$$|\langle m \rangle| \simeq \sqrt{\Delta m^2_{\mathrm{LSND}}} \simeq 0.45 - 1.4 \, \mathrm{eV} \qquad (\text{SMA-MSW}) \,. \tag{3.18}$$

Hence, the experimental upper bound (1.3) indicates that *the SMA-MSW solution of the solar neutrino problem is disfavored in scheme A.*

Furthermore, the upper bound $N^{\mathrm{BBN}}_\nu < 4$ for the effective number of neutrinos in Big-Bang Nucleosynthesis (BBN) (see, for example, [60]) implies that [54,57,58]

$$|U_{s1}|^2 + |U_{s2}|^2 \lesssim 10^{-2} \,, \tag{3.19}$$

in scheme A. The analysis of recent astrophysical data yields the upper bound[3] $N^{\mathrm{BBN}}_\nu \leq 3.2$ at 95% CL [61], although the issue is still rather controversial (see [62,63]). The inequalities (3.10) and (3.19), together with the unitarity of the mixing matrix, imply that the oscillations of solar neutrinos occur mainly in the $\nu_e \to \nu_s$ channel [57,58]. In this case, the analysis of solar neutrino data in terms of two-generation $\nu_e \to \nu_s$ oscillations is valid in the four-neutrino scheme A if the usual two-generation mixing parameters Δm^2 and ϑ are identified, respectively, with $\Delta m^2_{\mathrm{sun}} = \Delta m^2_{43}$ and ϑ_{sun} defined in Eq. (3.16) (from Eqs. (3.10), (3.16), (3.19) and the unitarity of the mixing matrix we obtain $|U_{s3}|^2 \simeq \sin^2 \vartheta_{\mathrm{sun}}$

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[3] The bound $N^{\mathrm{BBN}}_\nu \leq 3.2$ [61] implies that $|U_{s1}|^2 + |U_{s2}|^2 \lesssim 5 \times 10^{-4}$ [57,58] in scheme A!

and $|U_{s4}|^2 \simeq \cos^2 \vartheta_{\text{sun}})$. The results of the analyses of solar neutrino data in terms of two-generation $\nu_e \to \nu_s$ oscillations show that only the SMA-MSW solution is allowed [38,42,43]. Therefore, comparing Eqs. (1.3) and (3.18) we conclude that *the scheme A is disfavored by the experimental upper bound $|\langle m \rangle|$ and the BBN bound $N_\nu^{\text{BBN}} < 4$.*

Summarizing, the data from oscillation experiments, from neutrinoless double-β decay experiments and from the measurement of the abundances of primordial elements indicate that, *among all the possible four-neutrino schemes there is only one allowed, scheme B.*

B. Scheme B

In scheme B, the BBN upper bound $N_\nu^{\text{BBN}} < 4$ implies that [57,58]

$$|U_{s3}|^2 + |U_{s4}|^2 \lesssim 10^{-4} \,. \tag{3.20}$$

From this inequality, Eq. (3.11) and the unitarity of the mixing matrix it follows that the oscillations of solar neutrinos occur mainly in the $\nu_e \to \nu_s$ channel. Therefore, the analysis of solar neutrino data in terms of two-generation $\nu_e \to \nu_s$ oscillations is valid in the four-neutrino scheme B if the usual two-generation mixing parameters Δm^2 and ϑ are identified, respectively, with $\Delta m^2_{\text{sun}} = \Delta m^2_{21}$ and ϑ_{sun} defined by

$$|U_{e1}|^2 \simeq \cos^2 \vartheta_{\text{sun}} \,, \qquad |U_{e2}|^2 \simeq \sin^2 \vartheta_{\text{sun}} \tag{3.21}$$

(from Eqs. (3.11), (3.20), (3.21) and the unitarity of the mixing matrix we obtain $|U_{s1}|^2 \simeq \sin^2 \vartheta_{\text{sun}}$ and $|U_{s2}|^2 \simeq \cos^2 \vartheta_{\text{sun}}$). Since the results of the analyses of solar neutrino data in terms of two-generation $\nu_e \to \nu_s$ oscillations [38,42,43] show that only the SMA-MSW solution is allowed, with $10^{-3} \lesssim \sin^2 2\vartheta_{\text{sun}} \lesssim 10^{-2}$, we have $2.5 \times 10^{-4} \lesssim |U_{e2}|^2 \lesssim 2.5 \times 10^{-3}$. Therefore, in scheme B we have

$$|U_{e1}|^2 \simeq 1 \,, \qquad |U_{e2}|^2 \,, |U_{e3}|^2 \,, |U_{e4}|^2 \ll 1 \,. \tag{3.22}$$

In the scheme B there are two possibilities: a quasi-degenerate mass spectrum

$$\underbrace{\overbrace{m_1 < m_2}^{\text{sun}} \lesssim \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{LSND}} \qquad \text{(BD)} \tag{3.23}$$

or a mass hierarchy

$$\underbrace{\overbrace{m_1 < m_2}^{\text{sun}} \ll \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{LSND}} \qquad \text{(BH)} \tag{3.24}$$

If the quasi-degenerate mass spectrum BD is realized in nature, it is clear that from Eqs. (1.1) and (3.22) we have

$$|\langle m \rangle| \simeq m_1 \,. \tag{3.25}$$

In this case, the experimental upper bound (1.3) implies that

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$$m_1 \lesssim 0.2 - 0.4 \, \text{eV} \,. \tag{3.26}$$

The observation of neutrinoless double-β decay by the next generation of experiments, which are sensitive to values of $|\langle m \rangle|$ in the range $10^{-2}-10^{-1}$ eV [17], together with the confirmation of the four-neutrino scheme B by neutrino oscillation experiments, will provide an evidence in favor of the quasi-degenerate scheme BD.

If the hierarchical mass spectrum BH is realized in nature, the absence of unlikely fine-tuned cancellations between the contributions of m_1, m_2 and m_3, m_4 to the effective Majorana mass (1.1) implies that

$$|\langle m \rangle| \simeq \max[|\langle m \rangle|_{12}, |\langle m \rangle|_{34}] \,, \tag{3.27}$$

with

$$|\langle m \rangle|_{12} \equiv \left| U_{e1}^2 \, m_1 + U_{e2}^2 \, m_2 \right| \,, \tag{3.28}$$
$$|\langle m \rangle|_{34} \equiv \left| U_{e3}^2 \, m_3 + U_{e4}^2 \, m_4 \right| \,. \tag{3.29}$$

From Eq. (3.22), if $m_1 \simeq m_2$ we have $|\langle m \rangle|_{12} \simeq m_2$ and the contribution of $|\langle m \rangle|_{12}$ to $|\langle m \rangle|$ could be sizable. On the other hand, if $m_1 \ll m_2$ we have $|\langle m \rangle|_{12} \ll m_2$ and the contribution of $|\langle m \rangle|_{12}$ to $|\langle m \rangle|$ is strongly suppressed. In this case, if there are no unlikely fine-tuned cancellations between the contributions of m_1 and m_2 to $|\langle m \rangle|_{12}$, we have $|\langle m \rangle|_{12} \simeq |\langle m \rangle|_2$ with $|\langle m \rangle|_2$ in the range (2.11). In any case, at present it is not known if $m_1 \simeq m_2$ or $m_1 \ll m_2$ and we cannot infer the contribution of $|\langle m \rangle|_{12}$ to $|\langle m \rangle|$.

Let us consider now the contribution $|\langle m \rangle|_{34}$ to the effective Majorana mass. In principle it is possible that $|\langle m \rangle|_{34} = 0$ if $U_{e3}^2 \, m_3 + U_{e4}^2 \, m_4 = 0$, *i.e.*

$$|U_{e3}|^2 \, m_3 = |U_{e4}|^2 \, m_4 \tag{3.30}$$

and

$$|\arg(U_{e3}) - \arg(U_{e4})| = \pi/2 \tag{3.31}$$

(the condition (3.31) is satisfied if CP is conserved and ν_3 and ν_4 have opposite CP parities [64,24,2,27]). However, even if $m_3 \simeq m_4$, since $|U_{e3}|^2 + |U_{e4}|^2 \ll 1$ (see Eq. (3.11)), there is no reason to have $|U_{e3}|^2 \simeq |U_{e4}|^2$. On the other hand, the explanation of the atmospheric neutrino data with $\nu_\mu \to \nu_\tau$ oscillations [9–13], that is favored by the latest Super-Kamiokande data [65], requires a large mixing in the ν_μ, ν_τ–ν_3, ν_4 sector which could be related to the fact that $|U_{\mu3}|^2 + |U_{\mu4}|^2$ and $|U_{\tau3}|^2 + |U_{\tau4}|^2$ are large (close to one) and $m_3 \simeq m_4$.

Therefore, in the following we will assume that $|U_{e3}|^2$ and $|U_{e4}|^2$ have different orders of magnitude. In this case, the contribution of m_3 and m_4 to the effective Majorana mass is given by

$$|\langle m \rangle|_{34} \simeq d_e \sqrt{\Delta m_{\text{LSND}}^2} \,, \tag{3.32}$$

where we have taken into account that $m_3 \simeq m_4 \simeq \sqrt{\Delta m_{41}^2} = \sqrt{\Delta m_{\text{LSND}}^2}$ and we have defined

$$d_\alpha \equiv \sum_{k=3,4} |U_{\alpha k}|^2 \qquad (\alpha = e, \mu, \tau, s) \,. \tag{3.33}$$

It has been shown in [55] that d_e is small in scheme B:

$$d_e \leq a_e^{\text{Bugey}} \,, \tag{3.34}$$

with

$$a_e^{\text{Bugey}} = \frac{1}{2} \left(1 - \sqrt{1 - \sin^2 2\vartheta_{\text{Bugey}}} \right) \,. \tag{3.35}$$

Here $\sin^2 2\vartheta_{\text{Bugey}}$ is the upper value of the two-neutrino mixing parameter $\sin^2 2\vartheta$ obtained from the Bugey exclusion curve [51] as a function of $\Delta m^2 = \Delta m^2_{\text{LSND}}$, where Δm^2 is the two-neutrino mass-squared difference used in the analysis of the Bugey data (the upper bound (3.11) has been obtained from the inequality (3.34) restricting Δm^2_{LSND} in the LSND-allowed range (3.4)).

From Eqs. (3.32) and (3.34), for $|\langle m \rangle|_{34}$ we obtain the upper bound [25–27]

$$|\langle m \rangle|_{34} \lesssim a_e^{\text{Bugey}} \sqrt{\Delta m^2_{\text{LSND}}} \,, \tag{3.36}$$

The numerical value of this upper bound as a function of Δm^2_{LSND} is depicted by the solid line in Fig. 2. The dashed line in Fig. 2 represents the unitarity limit $|\langle m \rangle|_{34} \leq \sqrt{\Delta m^2_{\text{LSND}}}$.

The amplitude $A_{\mu e} = 4 \left| \sum_{k=3,4} U_{ek} U_{\mu k}^* \right|^2$ of short-baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations in scheme B is bounded by [55]

$$A_{\mu e} \leq 4 \, d_e \, d_\mu \,. \tag{3.37}$$

Since d_μ is large in scheme B [55], we have

$$d_e \geq \frac{A_{\mu e}^{\text{min}}}{4} \,, \tag{3.38}$$

where $A_{\mu e}^{\text{min}}$ is the minimum value of $A_{\mu e}$ measured in the LSND experiment. The physical reason of this lower bound for d_e is that ν_e must have some mixing with ν_3 and/or ν_4 in order to generate the oscillations observed in the LSND experiment.

Taking into account Eq. (3.32), the inequality (3.38) leads to the lower bound

$$|\langle m \rangle|_{34} \gtrsim \frac{A_{\mu e}^{\text{min}}}{4} \sqrt{\Delta m^2_{\text{LSND}}} \,. \tag{3.39}$$

The numerical value of this lower bound as a function of Δm^2_{LSND} is shown in Fig. 2 by the dotted curve that, together with the solid line obtained from the upper bound (3.36) defines an allowed region in the Δm^2_{LSND}–$|\langle m \rangle|_{34}$ plane (shadowed area). From Fig. 2 one can see that

$$6.9 \times 10^{-4} \, \text{eV} \lesssim |\langle m \rangle|_{34} \lesssim 2.1 \times 10^{-2} \, \text{eV} \,. \tag{3.40}$$

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Summarizing, in the framework of the scheme BH in Eq. (3.24) we have made three assumptions: (i) massive neutrinos are Majorana particles, (ii) there is no unlikely fine-tuned cancellations between the contributions of m_1, m_2 and m_3, m_4 to the effective Majorana mass $|\langle m \rangle|$, (iii) the two small elements U_{e3} and U_{e4} of the neutrino mixing matrix have different orders of magnitude. Under these reasonable assumptions, we have obtained the following allowed range for the effective Majorana mass in $\beta\beta_{0\nu}$ decay:

$$7 \times 10^{-4}\,\mathrm{eV} \lesssim |\langle m \rangle| \lesssim 2 \times 10^{-2}\,\mathrm{eV}\,, \tag{3.41}$$

from the contribution of m_3 and m_4 alone (the effective Majorana mass could be even larger than $2 \times 10^{-2}\,\mathrm{eV}$ if $m_1 \simeq m_2 \gtrsim 2 \times 10^{-2}\,\mathrm{eV}$). Such values of the effective Majorana mass could be measured by future $\beta\beta_{0\nu}$ decay experiments [17,19].

IV. CONCLUSIONS

We have derived lower limits for the the effective Majorana mass in neutrinoless double-β decay in the scheme with mixing of three neutrinos and a mass hierarchy [Eq. (2.2)] under the natural assumptions that massive neutrinos are Majorana particles and there are no large cancellations among the contributions of the different neutrino masses. If there is a hierarchy of neutrino masses, large cancellations are unlikely (unless an unknown symmetry is at work), because they require a fine-tuning among the values of the neutrino masses and the elements of the neutrino mixing matrix, which are independent quantities.

Under the only assumption that massive neutrinos are Majorana particles, we have shown that, among all the possible four-neutrino schemes that can accommodate the results of solar and atmospheric experiments and the results of the LSND experiment, only the scheme B [Eq. (3.9)] is compatible with the experimental results on neutrinoless double-β decay and the measurements of the cosmic abundances of elements produced in Big-Bang Nucleosynthesis. In the scheme B there are two possibilities: the quasi-degenerate mass spectrum BD [Eq. (3.23)] and the hierarchical mass spectrum BH [Eq. (3.24)].

If the quasi-degenerate four-neutrino scheme BD is realized in nature, neutrinoless double-β decay should be observed by the next generation of experiments, which will be sensitive to $|\langle m \rangle| \sim 10^{-2} - 10^{-1}$ eV.

In the framework of the hierarchical four-neutrino scheme BH, we have shown that there is a lower bound for the effective Majorana mass in $\beta\beta_{0\nu}$ decay, under the assumptions that massive neutrinos are Majorana particles, there are no large cancellations among the contributions of m_1, m_2 and m_3, m_4 and the two small elements U_{e3} and U_{e4} of the neutrino mixing matrix have different orders of magnitude.

We hope that the indications presented here in favor of a lower bound, albeit small, for the effective Majorana mass in neutrinoless double-β decay will encourage the development of future $\beta\beta_{0\nu}$ experiments.

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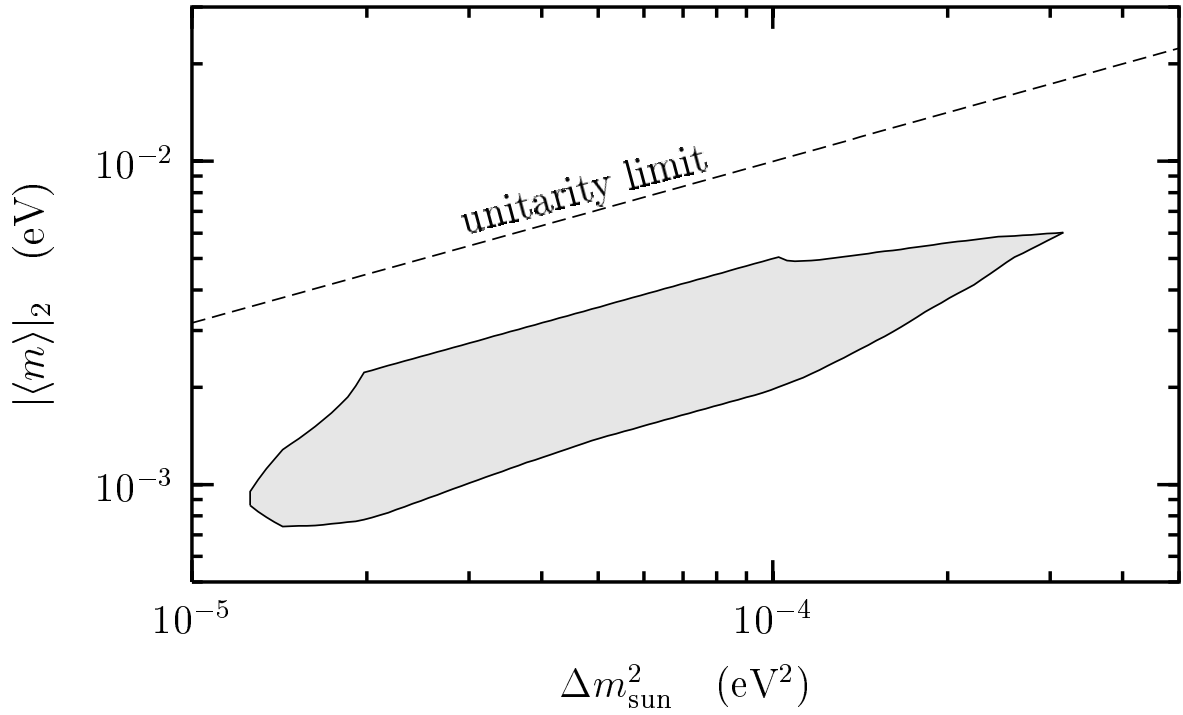
FIG. 1. The shadowed area shows the allowed range for $|\langle m \rangle|_2$ as a function of Δm_{sun}^2 in the scheme with mixing of three neutrinos with a mass hierarchy discussed in Section II, in the case of the LMA-MSW solution of the solar neutrino problem. The shadowed area has been obtained using Eq. (2.6) and the allowed range for $\sin^2 2\vartheta_{\text{sun}}$ given by the LMA-MSW region (99% CL) in the $\sin^2 2\vartheta_{\text{sun}}$–$\Delta m_{\text{sun}}^2$ plane presented in Fig. 2 of Ref. [42]. The dashed line represents the unitarity limit $|\langle m \rangle|_2 \leq \sqrt{\Delta m_{\text{sun}}^2}$.
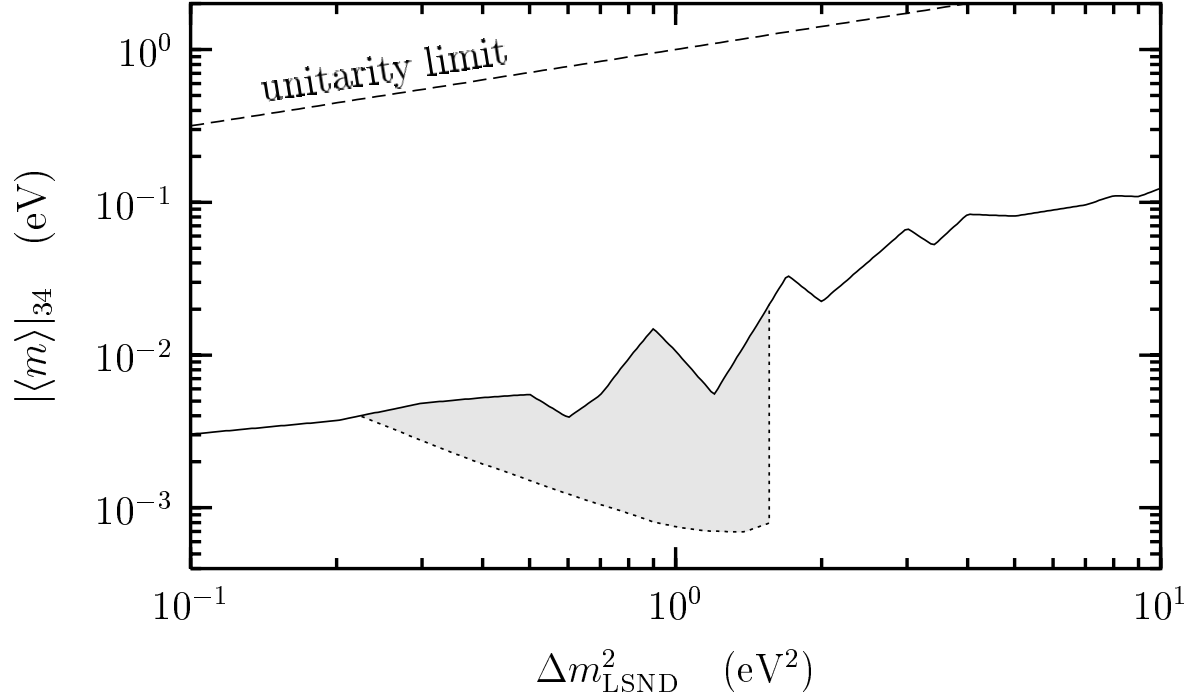
FIG. 2. The shadowed region shows the allowed range for $|\langle m \rangle|_{34}$ as a function of $\Delta m^2_{\mathrm{LSND}}$ in the four-neutrino scheme BH (see Eq. (3.24)). The solid line represents the upper bound in Eq. (3.36) and the dotted line represents the lower bound in Eq. (3.39). The dashed line represents the unitarity limit $|\langle m \rangle|_{34} \leq \sqrt{\Delta m^2_{\mathrm{LSND}}}$.